Muller's method

1. Use three steps of Muller's method to approximate a root of the function $f(x) \overset{\text{def}}{=} \frac{\sin(x)}{x} + e^{-x}$ starting with $x_0 = 3.0$. $x_1 = 3.1$ and $x_2 = 3.2$.

Answer: To ten significant digits, we have 3.0, 3.1, 3.2, 3.266500437, 3.266500387, 3.266379463

2. Use one step of Muller's method to approximate a root of the function $f(x) \overset{\text{def}}{=} x^3 - 3x + 1$ starting with $x_0 = -1.5$, $x_1 = -0.3$ and $x_2 = 1.8$. What happened?

Answer: The three points happen to be co-linear, and therefore the interpolating quadratic happens to be a linear function, which has a root of 181/21 or approximately 8.619047619047619.

3. What is the cause for the sequence of approximations in Question 2?

Answer: Muller's method works best if we are close to a root, as opposed to three arbitrary points.

4. Use one step of Muller's method to approximate a root of the function $f(x) \overset{\text{def}}{=} x^3 - 3x + 1$ starting with $x_0 = -1.4$, $x_1 = -0.3$ and $x_2 = 1.8$. What happened?

Answer: The interpolating quadratic has no roots as the discriminant is negative.

5. What is the cause for the sequence of approximations in Question 4?

Answer: Again, Muller's method works best if we are close to a root, as opposed to three arbitrary points.

6. Use three steps of Muller's method to approximate a root of the function $f(x) \overset{\text{def}}{=} x^3 - 3x + 1$ starting with $x_0 = 1.4$. $x_1 = 1.5$ and $x_2 = 1.6$.

Answer: Recalling that we reorder the values, we have 1.4, 1.6, 1.5, 1.532017786, 1.532088848, 1.532088886.

7. If you continued to iterate Muller's method in Question 6, what root does it converge to?

Answer: To ten significant digits, 1.532088886; that is, what is already the last value.

8. In general, should you apply Muller's method if you don't already have an idea as to what a root of a function is?

Answer: In general, no. Muller's method is a tool to refine an approximation of a root, not to check if a function has a root. If you start with an arbitrary initial point, it may or may not converge to a root if there is one, so non-convergence does not suggest there is no root.

9. In general, should you apply Muller's method if you don't already have an idea as to what a root of a polynomial is?

Answer: In this case, in general, the answer is yes: it will converge to a root, even if that root is complex, so long as the system supports complex arithmetic. In this case, a single root can be found, and that root can then be factored out, allowing one to find the remaining roots.